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International Journal of Heat and Mass Transfer 44 (2001) 3233–3240

International Journal of HEAT and MASS TRANSFER

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Condensation of vapours on a fin in the presence of non-condensable gas

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Received 15 March 1998; received in revised form 25 October 2000

Abstract

Condensation of vapours on a vertical fin in the presence of non-condensable gas is formulated and solved for a wide range of system parameters, making use of the analogy between heat and mass transfer. The formulation enables the derivation of special cases such as condensation of vapours on a vertical isothermal surface with and without non-condensable gas in the vapour. Besides, the analysis of condensation of vapour on a fin has revealed non-isothermal conditions of the thin fin with the temperature of the liquid–vapour interface also changing along the length of the fin. The numerical results from the analysis are rendered into dimensionless equations that can be used to predict the mean condensation heat transfer coefficients. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Nusselt [5] first investigated condensation of pure vapours on a vertical surface within the framework of numerous assumptions. Subsequently, many investigators improved the Nusselt analysis and the limitations imposed by the assumptions of Nusselt were lifted to an extent. The review articles of by Merte [4] have presented the details of such investigations. Recently, the importance of fins as extended surfaces is recognised as one of the most efficient methods to facilitate heat dissipation. Besides, condensation on fins is considered as an efficient technique when employed in the condenser section of heat pipes.

Minkowycz and Sparrow [3] studied condensation of vapours in the presence of non-condensable vapours. Their analysis includes the effects of interfacial resistance, superheat of vapour, free convection associated with both temperature and concentrated gradients, physical property dependence on temperature variation on condensation heat transfer phenomenon. Kroger and Roshenow [2] investigated condensation of vapours of potassium in the presence of Argon and Helium. Sparrow and Eckert [10] solved the problem of effect of superheated vapour and non-condensable gases on laminar film condensation. Rose [6] presented an approximate analysis for forced convection condensation in the presence of a non-condensing gas. Chao-yang Wang and Chaun-jing Tu [1] found that about 50% reduction in heat transfer coefficient is noticed even for 0.5% of non-condensable gas in a porous medium. Tarahiro Mamyoda and Kichi Asano [11] studied condensation of vapours in the presence of non-condensable gas on a short horizontal tube with the external flow of vapour. They compared their analysis with their own experimental data. The agreement between the two was found to be satisfactory.

Sarma and co-workers [7,8] solved the problem of condensation of vapours on a fin of variable thickness and also on a porous metal fin. However, their analyses are only limited to the case of condensation of pure vapours. In reality, heat pipes though evacuated before charging them with the working medium still contain

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PII: S0017-9310(00)00346-X

Nomenclature

A	cross-sectional area of the fin, m^2
(A_0–A_4)	constants in quartic polynomial (see Eq. (3))
A^*	dimensionless heat capacity parameter (see Eq. (25))
C_p	specific heat at constant temperature, kJ/kg K
D	diffusion coefficient, m^2/s
g	acceleration due to gravity, m/s^2
Gr_l	Grashof number, gL^3/ν_l^2
h	local heat transfer coefficient, W/m^2 K
h_m	mean heat transfer coefficient, W/m^2 K
H_m	mass transfer parameter (see Eq. (15))
h_{fg}	latent heat of vapourisation, kJ/kg
k_l	liquid thermal conductivity, W/m K
k_w	fin thermal conductivity, W/m K
L	length of the fin, m
m_l	mass flow rate of condensate, kg/s/m
M^*	fin parameter
Nu_c	local Nusselt number, hL/k_l
Nu_m	average Nusselt number
φ	augmentation factor (Nu_{fin}/Nu_{iso} without non-condensable gas in vapour)
P	total pressure of vapour, N/m^2
P_v	partial pressure of vapour, N/m^2
Pr_l	Prandtl number of saturated liquid
Re_l	film Reynolds number of saturated liquid
R_v	gas constant of the vapour kJ/kg/K
S	perimeter of the fin, m
Sh	Sherwood number
T_s	saturation temperature, K
T_w^+	dimensionless wall temperature, T_w/T_s
T_i^+	dimensionless interfacial temperature, T_i/T_s
x, y	Cartesian co-ordinates (refer to Fig. 1)

Greek symbols

δ_c	condensate film thickness, m
Δ	gaseous film thickness, m
η	dimensionless space variable, x/L
μ	dynamic viscosity of condensate, kg/m s
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3

Subscripts

c	condensate
l	liquid
s	steam
v	vapour
w	wall
i	interfacial

some traces of non-condensable gases. The presence of a non-condensable makes the process of condensation of pure vapours inefficient and condensation heat transfer coefficient decreases remarkably due to the presence of a non-condensable component such as air. Such a problem for the case of a fin does not appear to have been solved so far in the literature. The purpose of this article is to formulate and solve the problem of condensation of vapours in the presence of a non-condensable gas. In addition, the numerical results are rendered into dimensionless equations consisting of system parameters so that such equations can be employed in design.

2. Formulation

The lateral face of a thin fin is exposed to a medium of vapours with traces of non-condensables present in the ambient quiescent vapours. The configuration and the co-ordinate system of the fin are shown in Fig 1. The total pressure P of the ambient medium constitutes two components, one due to the vapour $P_{v\infty}$ and the other due to the gas $P_{g\infty}$ such that $P = P_{v\infty} + P_{g\infty}$. The base of the fin is maintained at a temperature that is far less than the saturation temperature of vapour corresponding to the system pressure. The condensation on the lateral face of the fin is dependent on the thermal gradients existing along the fin length and the process of mass diffusion of vapours through the gas film formed adjacent to the condensate film. The gas film formed adjacent to the condensate film offers additional resistance, and the diffusion of vapour molecules is dependent on the partial pressure potential of the vapour across the film. It is assumed that the interface temperature corresponds to the partial pressure of vapour P_{vi} that prevails at the interface of the condensate. In the formulation of the problem, the following assumptions

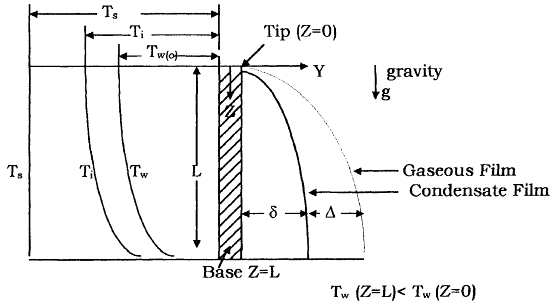

Fig. 1. Physical model of fin and co-ordinate system.

are made in addition to some of the assumptions employed by Nusselt [5] in his pioneering analysis.

The fin is thin and thermal conduction along the length of the fin facilities existence of thermal gradients with the temperature decreasing towards the base of the fin.

- Further, the condensation process occurs at the interface at a temperature that happens to be the saturation temperature corresponding to the partial pressure different from the total pressure of the system at distances far away from the lateral face of the fin.
- The molecular diffusion of the vapour across the gas film at any location is given by the modified form of Fick's law, i.e., for the co-ordinate system shown in Fig. 1 as

$$\frac{1}{P - P_{\mathrm{v\,i}}}\left[\frac{\mathrm{d}P_{\mathrm{v}}}{\mathrm{d}y}\right]_{y=0} = \frac{W_{\mathrm{v}}R_{\mathrm{v}}T_{\mathrm{av}}}{DP} \tag{1}$$

with the y-ordinate measured normal to the condensate film. Eq. (1) is valid for isothermal conditions. However, the non-isothermal conditions prevailing across the boundary layer is being approximated by considering $T_{\mathrm{av}} = 0.5(T_{\mathrm{i}} + T_{\mathrm{s}})$. The partial pressure distribution at any location of the fin is given by the parabolic distribution

$$\frac{P_{\mathrm{v}\infty} - P_{\mathrm{v}}}{P_{\mathrm{v}\infty} - P_{\mathrm{v\,i}}} = \left(1 - \frac{y}{\Delta}\right)^2. \tag{2}$$

The concentration boundary layer is of the same magnitude as the hydrodynamic boundary layer for $Sc = 0.8$. Further, the analogy between heat and mass transfer is valid.

It is assumed that the thermodynamic parameters, pressure and saturation temperature, i.e., (P, T) can be represented by a quartic polynomial for any given species of a vapour. For example, for the case of steam, the following relationship can be obtained:

$$\frac{P_{\mathrm{i}}}{P} = \phi\left(\frac{T_{\mathrm{i}}}{T_{\mathrm{s}}}\right) = A_0 + A_1\left(\frac{T_{\mathrm{i}}}{T_{\mathrm{s}}}\right) + A_2\left(\frac{T_{\mathrm{i}}}{T_{\mathrm{s}}}\right)^2 + A_3\left(\frac{T_{\mathrm{i}}}{T_{\mathrm{s}}}\right)^3 + A_4\left(\frac{T_{\mathrm{i}}}{T_{\mathrm{s}}}\right)^4, \tag{3}$$

where the coefficients (A_0, A_4) can be obtained for the case when $T_{\mathrm{s}} = 373$ K, $P = 1$ bar, $T_{\mathrm{i}} \leqslant 373$ K, $P_{\mathrm{i}} \leqslant P_{\mathrm{v}\infty}$, $A_0 = 67.447$, $A_1 = -348.132$, $A_2 = 677.555$, $A_3 = -589.884$, $A_4 = 194.014$. Besides, the perfect gas equation is assumed to be applicable to vapour, i.e.,

$$P_{\mathrm{v}} = \rho_{\mathrm{v}}R_{\mathrm{v}}T_{\mathrm{v}}. \tag{4}$$

Employing these assumptions, for the configuration and the co-ordinate system shown in Fig. 1 Nusselt's analysis for thin laminar condensate films yields the following relationship:

$$\frac{\mathrm{d}Re_{\mathrm{l}}^{4/3}}{\mathrm{d}\eta} = 5.87A^*\left[T_{\mathrm{i}}^+ - T_{\mathrm{w}}^+\right], \tag{5}$$

where

$$A^* = \frac{C_{\mathrm{p}}T_{\mathrm{s}}}{h_{\mathrm{fg}}\,Pr_{\mathrm{l}}}\,Gr_{\mathrm{l}}^{1/3},$$

$$T_{\mathrm{i}}^+ = \left(\frac{T_{\mathrm{i}}}{T_{\mathrm{s}}}\right), \qquad T_{\mathrm{w}}^+ = \left(\frac{T_{\mathrm{w}}}{T_{\mathrm{s}}}\right),$$

$$\eta = (Z/L),$$

$$Re_{\mathrm{l}} = \left(\frac{4m_{\mathrm{l}}}{\mu_{\mathrm{l}}}\right), \qquad Gr_{\mathrm{l}} = \left(\frac{gL^3}{\nu_{\mathrm{l}}^2}\right),$$

$$m_{\mathrm{l}} = \rho_{\mathrm{l}}\int_0^{\delta} u\,\mathrm{d}y.$$

The thermal conduction in the thin fin is given by the following equation in dimensionless form:

$$\frac{\mathrm{d}^2T_{\mathrm{w}}^+}{\mathrm{d}\eta^2} = -M^*\frac{\left[T_{\mathrm{i}}^+ - T_{\mathrm{w}}^+\right]}{Re_{\mathrm{l}}^{1/3}}, \tag{6}$$

where

$$M^*\ \text{(fin parameter)} = \left(\frac{K_{\mathrm{l}}SL}{K_{\mathrm{w}}A}\,Gr_{\mathrm{l}}^{1/3}\right).$$

For a given geometry of fin and system pressure, the fin parameter physically signifies the ratio of thermal conductivity of the condensate to that of fin material, respectively.

Eq. (6) is to be solved subject to thermal conditions prevailing at the tip and the base of the fin. For short fins the boundary conditions can be written as

$$\begin{aligned} &\eta = 0, \quad \frac{\mathrm{d}T_{\mathrm{w}}^+}{\mathrm{d}\eta} = 0, \quad Re_{\mathrm{l}} = 0,\\ &\eta = 1, \quad T_{\mathrm{w}}^+ = T_{\mathrm{w}}^+(1)\ \text{(to be prescribed)}. \end{aligned} \tag{7}$$

Eqs. (5) and (6) contain three independent variables T_{i}^+, T_{w}^+ and Re_{l}, which are functions of the space variable η. Hence, another equation is needed describing the diffusional process of the vapour molecules towards interface.

Based on the principle of analogy it can be shown that the concentration boundary layer thickness can be expressed from the following relationship for $Sc = 0.8$. The local value of Sherwood number is given by

$$Sh_x = 0.356\left[g\frac{\Delta\rho}{\rho_{\mathrm{v}\infty}}\frac{x^3}{\nu_{\mathrm{s}}^2}\right]^{1/4} \tag{8}$$

or

$$\frac{\Delta}{L} = 5.62\left(\frac{\nu_{\mathrm{v}}}{gL^3}\right)^{1/4}\left(\frac{\rho_{\mathrm{v}\infty}}{\Delta\rho_{\mathrm{v}}}\right)^{1/4}\eta^{1/4}, \tag{9}$$

where

$$\frac{\Delta\rho_{\mathrm{v}}}{\rho_{\mathrm{v}\infty}} = \left\{1 - \frac{\rho_{\mathrm{v\,i}}}{\rho_{\mathrm{v}\infty}}\right\}.$$

It is to be noted that Eq. (8), though valid for constant value of $\rho_{\mathrm{v\,i}}$, is applicable to the variable conditions of $\rho_{\mathrm{v\,i}}$ since the variation of the numerical coefficient is marginal both for constant mass flux conditions and fixed concentration at the interface. This can be inferred from the study of Sparrow and Gregg [9] related to the free convective heat transfer studies on non-isothermal surfaces.

The density-ratio term can be written as follows with the help of Eqs. (3) and (4):

$$\frac{\Delta\rho_{\mathrm{v}}}{\rho_{\mathrm{v}\infty}} = \left\{\frac{\phi(T_{\mathrm{i}}^{+})}{T_{\mathrm{i}}^{+}}\right\}, \tag{10}$$

where $\phi(T_{\mathrm{i}}^{+})$ is the parameter corresponding to Eq. (3) developed by regression applied to thermodynamic data of the given species of vapour.

Further, making use of Eqs. (1) and (2) it can be approximated that the rate of mass diffusion W_{v} of the vapour molecules per unit area towards interface is as follows:

$$W_{\mathrm{v}} = 4\frac{P_{\mathrm{v}\infty}D}{\Delta R_{\mathrm{v}}T_{\mathrm{s}}}\left[\frac{1}{T_{\mathrm{i}}^{+}+1}\right]. \tag{11}$$

However, at the vapour–condensate interface, the heat conducted across the condensate film is equal to the heat associated with the process of phase transformation, i.e., condensation.Thus,

$$W_{\mathrm{v}} = \left[\frac{K_{\mathrm{l}}T_{\mathrm{s}}(T_{\mathrm{i}}^{+} - T_{\mathrm{w}}^{+})}{\delta_{\mathrm{c}}h_{\mathrm{fg}}}\right]. \tag{12}$$

Hence, from Eqs. (11) and (12) it can be shown that

$$\frac{\delta_{\mathrm{c}}}{L} = 0.25\left(\frac{\Delta}{L}\right)\left[\frac{K_{\mathrm{l}}T_{\mathrm{s}}^2R_{\mathrm{v}}}{P_{\mathrm{v}\infty}h_{\mathrm{fg}}D}\right](T_{\mathrm{i}}^{+} - T_{\mathrm{w}}^{+})(1 + T_{\mathrm{i}}^{+}). \tag{13}$$

From Nusselt's analysis, one can obtain that at any location η

$$\frac{\delta_{\mathrm{c}}}{L} = \left(\frac{3}{4}\right)^{1/3}\left[\frac{Re_{\mathrm{l}}}{Gr_{\mathrm{l}}}\right]^{1/3}. \tag{14}$$

Thus, from Eqs. (9), (10), (13) and (14) it can be shown that

$$\frac{(T_{\mathrm{i}}^{+} - T_{\mathrm{w}}^{+})(1 + T_{\mathrm{i}}^{+})}{\left[1 - \frac{\phi(T_{\mathrm{i}}^{+})}{T_{\mathrm{i}}^{+}}\right]^{1/4}} = 0.647\left[\frac{Re_{\mathrm{l}}^{1/3}}{Gr_{\mathrm{l}}^{1/2}}\right]\left[\frac{\eta^{-1/4}}{H_{\mathrm{m}}}\right], \tag{15}$$

where

H_{m} (mass transfer parameter)

$$= \left[\frac{K_{\mathrm{l}}T_{\mathrm{s}}^2R_{\mathrm{v}}}{P_{\mathrm{v}\infty}h_{\mathrm{fg}}D}\right]\left(\frac{\nu_{\mathrm{v}}}{\nu_{\mathrm{l}}}\right)^{1/2}, \quad \text{where } P_{\mathrm{v}\infty} = P - P_{\mathrm{g}\infty}.$$

Eqs. (5), (6) and (15) complete the description of the problem from the mathematical point of view, allowing the evaluation of T_{i}^{+}, T_{w}^{+} and Re_{l} as functions of the independent variable η with due regard to the boundary conditions already mentioned.

3. Local and average heat transfer coefficients

The overall condensation heat transfer coefficient at any location can be obtained as follows:

$$h(T_{\mathrm{s}} - T_{\mathrm{w}}) = \frac{K_{\mathrm{l}}(T_{\mathrm{i}} - T_{\mathrm{w}})}{\delta_{\mathrm{c}}}. \tag{16}$$

Eq. (16) in dimensionless form can be written as follows:

$$\frac{Nu_{\mathrm{c}}}{Gr_{\mathrm{l}}^{1/3}} = \left(\frac{4}{3}\right)^{1/3}\left[\frac{T_{\mathrm{i}}^{+} - T_{\mathrm{w}}^{+}}{1 - T_{\mathrm{w}}^{+}}\right]\frac{1}{Re_{\mathrm{l}}^{1/3}}. \tag{17}$$

The average Nusselt, Nu_{m} number can be obtained as

$$\frac{Nu_{\mathrm{m}}}{Gr_{\mathrm{l}}^{1/3}} = \int_0^1 \frac{Nu_{\mathrm{c}}}{Gr_{\mathrm{l}}^{1/3}}\,\mathrm{d}\eta. \tag{18}$$

Thus, it can be seen that

$$\frac{Nu_{\mathrm{c}}}{Gr_{\mathrm{l}}^{1/3}} = F[\eta, A^*, M^*, H] \tag{19}$$

and

$$\frac{Nu_{\mathrm{m}}}{Gr_{\mathrm{l}}^{1/3}} = F[A^*, M^*, H]. \tag{19a}$$

In the design estimation of the area, A, at the base of the fin, the following approximation can be made use of: $Q = h_{\mathrm{m}}(T_{\mathrm{s}} - T_m)A$, where $T_{\mathrm{m}} = 0.5(T_{\mathrm{s}} + T_{\mathrm{w}}(Z = L))$. In fact, such a procedure would lead to overestimation of the area, A, which is generally acceptable since it yields a better factor of safety.

On inspection it can be seen that Eqs. (5) and (6) can be manipulated to derive a first-order differential equation as follows:

$$\frac{\mathrm{d}T_{\mathrm{w}}^{+}}{\mathrm{d}\eta} = -0.227\frac{M^*}{A^*}\,Re_{\mathrm{l}}. \tag{20}$$

In obtaining Eq. (20) the following boundary condition is made use of:

$$\text{at } \eta = 0, \quad \frac{\mathrm{d}T_{\mathrm{w}}^{+}}{\mathrm{d}\eta} = 0 \text{ (negligible heat loss at tip).} \tag{20a}$$

Thus, the simultaneous solution of Eqs. (5), (15) and (20) will give the required solutions, i.e., Eqs. (19) and (19a). Further, when $M^* \rightarrow 0$, $T_{\mathrm{w}}^{+} = \text{constant}$. The problem corresponds to the case of condensation of vapours on a vertical isothermal surface in the presence of non-condensables.

An iterative numerical method is employed in solving Eqs. (5), (15) and (20) by writing them down in finite-difference form for a prescribed value of the temperature at the base of the fin and other system parameters A^* and M^*. A^*, M^* and H can be physically termed, respectively, as phase transformation parameter, conduction-ratio parameter and mass diffusion parameter.

4. Results and discussion

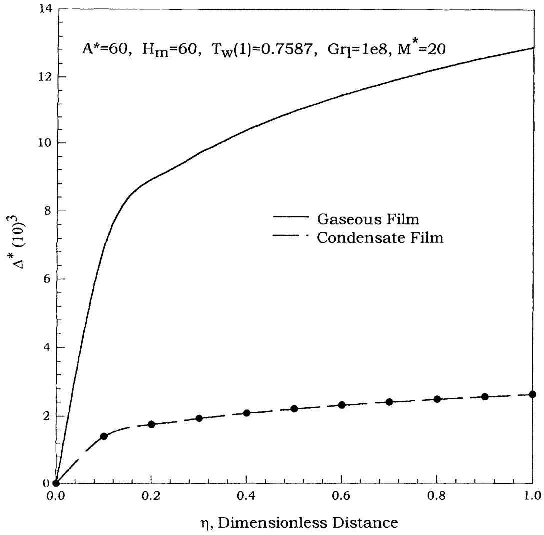

Fig. 3. Variation of film thickness along the fin.

In Fig. 2, the temperature variation of the fin and that of the interface of the condensate along its length are shown plotted for different values of the fin parameter M^* and other system conditions. The greater the value of the fin parameter the higher is the temperature difference between the tip and base of the fin. Further, it can be seen that the temperature of the condensate–vapour interface is also changing, indicating that the partial pressure variation of the interface is also decreasing monotonically; such an observation can also be made with the aid of Eq. (3). The solutions are accomplished for the case of negligible heat dissipation at the tip and a finite temperature at the base of the fin. The variation of the concentration boundary layer and the condensate film along the length of the fin are shown plotted in Fig. 3. Both the condensate and concentration boundary layers happen to vary monotonically along the length of the fin. Fig. 4 indicates that the thermal conductivity of the fin material can also influence the partial pressure of the vapour at the interface. It can be seen from Fig. 4 that the condensate film Reynolds number along the length of the film gradually increases for the case of condensation of pure vapours on a fin. It can be seen that an increase in thermal conductivity of the fin material will lead higher values of *Re* at any given location. The trends in the characteristics, i.e., nature of the gradient change with non-condensable gas present in the vapour as evident from Fig. 5. The film Reynolds

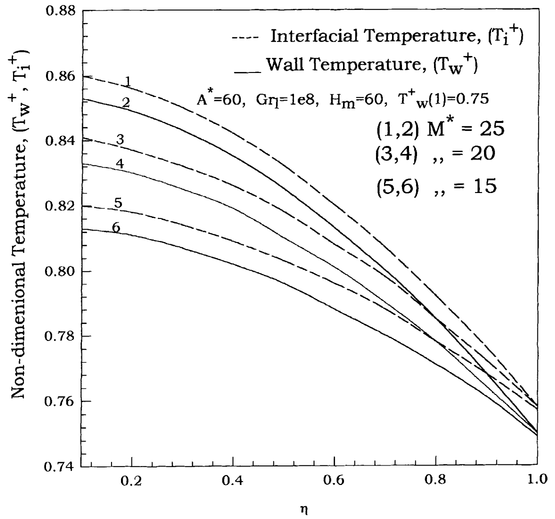

Fig. 2. Variation of wall and interfacial temperature of the condensate film along the fin.

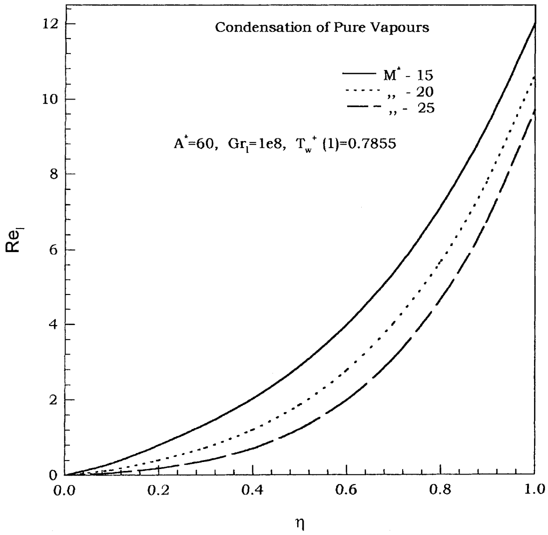

Fig. 4. Variation of film Reynolds along the fin for the case of condensation of pure vapours.

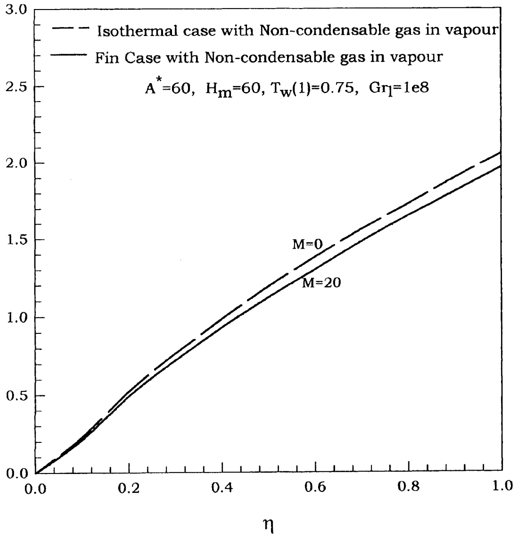

Fig. 5. Variation of film Reynolds in the presence of non-condensable gas.

number for the case of $M^* \to 0$ is always higher than those values for $M^* \ll 0$. $M^* \to 0$ physically signifies that the surface on which condensation takes place is under isothermal conditions. Another practical possibility is with regard to the different thermal boundary conditions that can be maintained both at the tip and base of the fin. One such case is that the two extreme ends of the fin can be maintained at two different temperatures. Fig. 6 represents the optimal values of M^* that should be chosen when the fin is supposed to operate between two different temperature limits as indicated in the legend of Fig. 6. Critical M^* is defined as the value obtained by iteration such that the tip and base temperatures of the fin are at specified values satisfying the negligible heatless condition at the tip. For the specified system conditions, the plot, i.e., Fig. 6 would enable us to choose the efficient material of the fin to be used in terms of its material conductivity.

The variation of local Nusselt along the fin for different values of M^* is shown plotted in Fig. 7. Nu_x increases with an increase in M^*. However, in the presence of a non-condensable gas, the local values of condensation heat transfer decrease substantially, as can be seen from the results of Fig. 8. Further, it can be inferred that a fin seems to be more effective than a plain isothermal surface represented by $M^* = 0$. The diminishment of the condensation heat transfer coefficient as a function of the mass transfer parameter H_m is shown in Fig. 9. The results reveal that with the content of non-condensable gas decreasing (i.e., H_m tending to a critical value) the results approach the case of condensation of pure vapours. Further, results of Fig. 9 indicate that a fin made out of material having better thermal conductivity gives better performance than the material having lower thermal conductivity. Although comparison of the present theory with experimental data could not be provided because of the lack of the data in the literature, several of the numerical results of the present analysis are rendered into dimensionless equations and presented in a form convenient for design calculations of fins on which condensation of vapours occurs with traces of non-condensable gases present in the medium (see Fig. 10). However, since the present analysis is so flexible a special case of condensation on a vertical surface in the

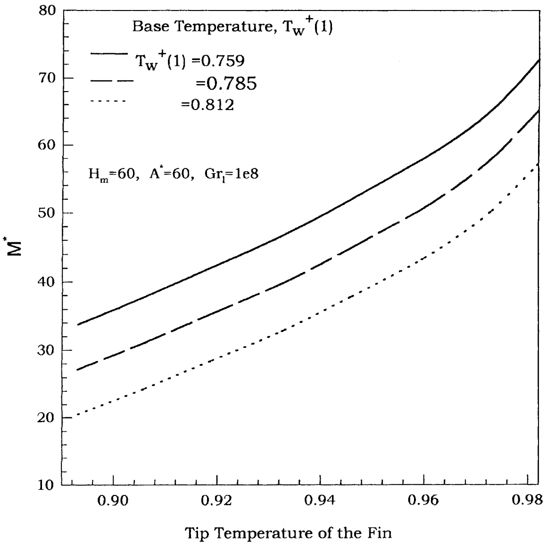

Fig. 6. Variation of critical fin parameter for different tip temperature and system conditions.

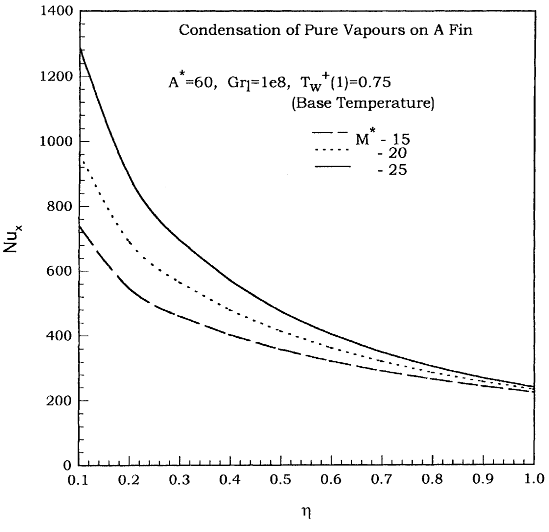

Fig. 7. Variation of local Nusselt number.

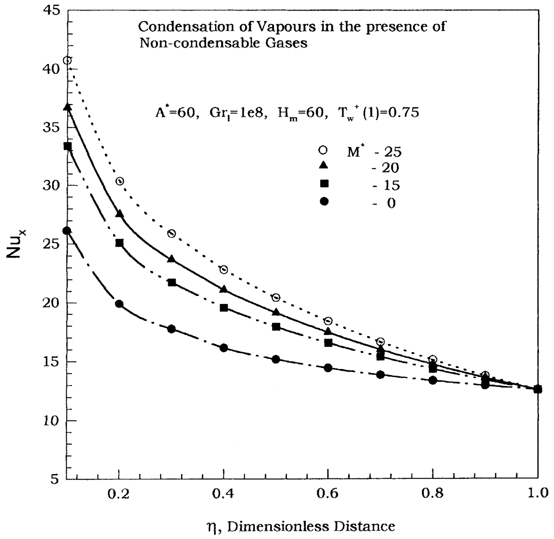

Fig. 8. Variation of local Nusselt number.

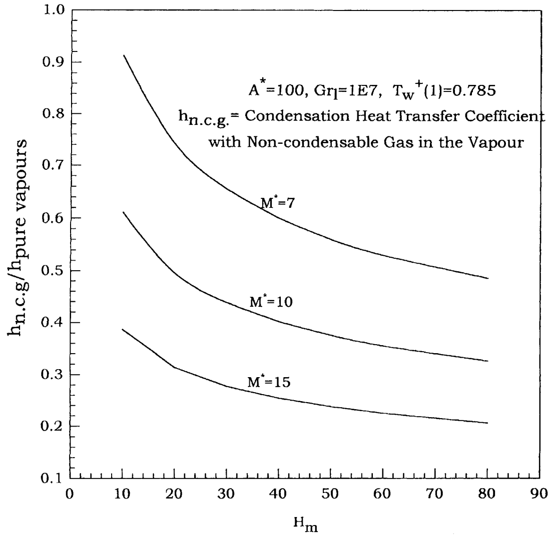

Fig. 9. Variation of heat transfer coefficient ratio with H_m (mass transfer parameter).

presence of non-condensable gases is solved for the system parameters as shown in the legend and the results taken from [2] are also shown in represented by lines A, B in Fig. 11. The trends in either case are similar.

However, the present analysis indicates rather rapid decrease of condensation heat transfer coefficient for very low concentration of non-condensables present in the condensing vapours. Thus, Eqs. (22) and (23) would serve as computational tools in the estimation of condensation heat transfer coefficients of vapour in the presence of non-condensable gases. It can be seen from Eqs. (21)–(25) that there is augmentation due to the condensation on a fin in relation to the phenomenon on an isothermal surface. Evidently condensation on a fin in the presence of non-condensable gas average yields an augmentation factor φ more than unity.

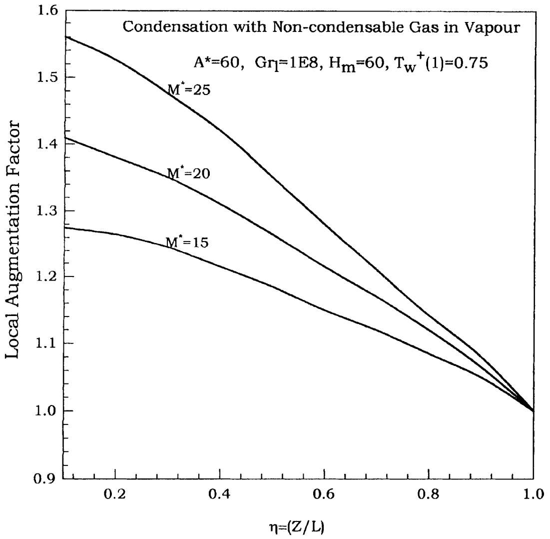

Fig. 10. Variation of local augmentation factor with fin parameter.

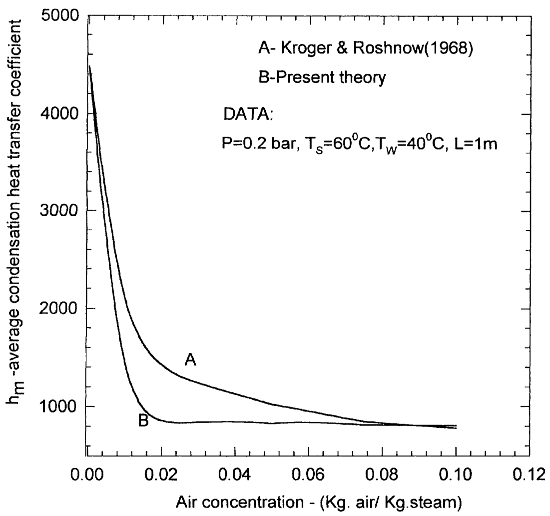

Fig. 11. Variation of condensation heat transfer coefficients with air concentration.

The following design equations are proposed:

1. Limiting or critical value of M^* when $0.785 < T_w^+(1) < 0.812$:

$$M^* = 6.25A^{*0.021}H_m^{0.9512}[1 - T_w^+(\eta = 1)]^{1.284}. \qquad (21)$$

2. Correlation for conditions with non-condensable gas and isothermal conditions of the wall:

$$\frac{Nu}{Gr_l^{1/3}} = \frac{0.6}{H_m^{0.9}[1 - T_w^+]^{0.845}}. \tag{22}$$

3. Correlation for condensation of pure vapours and wall maintained under isothermal conditions (Nusselt equation):

$$\frac{Nu}{Gr_l^{1/3}} = \left[\frac{0.943}{A^{*0.25}(1 - T_w^+)^{0.25}}\right]. \tag{23}$$

4. Correlation for the case of fin with non-condensable gas present in vapour:

$$Nu = \frac{7.537(A^*)^{0.01046}(Gr_l)^{0.3341}[1 - T_w^+(1)]^{0.5234}}{H_m^{0.3024}M^{*0.6892}}. \tag{24}$$

5. Correlation for condensation of pure vapours on a fin:

$$\frac{Nu}{Gr_l^{1/3}} = \frac{0.6775M^{*0.4369}}{A^{*0.3678}[1 - T_w^+(1)]^{0.3661}}, \tag{25}$$

where the parameters are defined as follows:

$$M^* = \frac{K_l SL\, Gr_l^{1/3}}{K_w A^*}, \quad A^* = \frac{C_p T_s}{h_{fg}} \frac{Gr_l^{1/3}}{Pr_l},$$

$$H_m = \frac{K_l T_s^2 R_v}{P_{v\infty} h_{fg} D}\left(\frac{v_s}{v_l}\right)^{1/2}, \quad T_w^+ = \frac{T_w}{T_s},$$

$$Gr_l = \frac{gL^3}{v_l^2}, \quad Pr_l = \frac{\mu_l C_p}{K_l}.$$

5. Conclusions

The following are the salient conclusions that can be arrived at from the theoretical study undertaken.

1. Condensation of vapours on the lateral face of the fin is distinctly different from the condensation process on a plain isothermal surface. According to the formulation, the partial pressure of the condensing vapour at the vapour–liquid interface monotonically varies from the tip to the base of the fin.
2. The formulation is made so flexible such that special cases can be derived. For example, the condensation process on a plain isothermal surface in the presence of a non-condensable gas can be obtained by choosing the fin parameter $M^* = 0$.
3. Condensation on a fin in the presence of non-condensable gas is found to be more effective in the sense that the magnitudes of local heat transfer coefficients are more than the values that can be anticipated on a plain isothermal surface.
4. Selection of the fin material can be made from the optimal solutions, i.e., Eq. (21) obtained with the boundary conditions, viz.,

$$\left.\frac{dT^+}{d\eta}\right|_{\eta=0} = 0$$

and

$T^+(1) = T_0^+(1)$ prescribed value at the base.

5. Eqs. (21)–(25) can be employed for design purposes for various systems that are likely to be encountered in practice.

Acknowledgements

Prof. Sarma and Prof. Sadik Kakac greatly appreciate the support received from the Department of Science and Technology, Government of India, New Delhi, and the National Science Foundation, Washington, USA.

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